Entropy – Quantifying Energy Degradation

Review - First Law

How to relate changes to the cause

Properties will change indicating change of state

Mass in

W_i

Q_in

Dynamic Energies as causes (agents) of change

Mass out

W_o

Q_out

Energy Balance

E_in – E_out = ∆E_sys, kJ or
e_in – e_out = ∆e_sys, kJ/kg or

W – E = ∆E_sys, kW

Review - First Law

Energy Balance – Control Volume

Steady-Flow

Steady-flow is a flow where all properties within boundary of the system remains constant with time

∆E_sys = 0, kJ; ∆e_sys = 0 , kJ/kg; ∆V_sys = 0, m^3; 
∆m_sys = 0 or m_in = m_out, kg 
∆m_sys = 0, kg/s

m_in – m_out = 0 or m_in = m_out, kg/s

Review - First Law

Quotes

- “The principal goal of education is to create men and women who are capable of doing new things, not simply repeating what other generations have done” – Jean Piaget

- “What we have to learn to do, we learn by doing” – Einstein
Review - First Law

**Mass & Energy Balance—Steady-Flow: Single Stream**

- **Mass balance**
  \[ \dot{m}_{\text{sys}} = 0, \quad \dot{m}_{\text{in}} = \dot{m}_{\text{out}}, \quad \text{kg/s} \]

- **Energy balance**
  \[ \Delta \dot{E}_{\text{sys}} = 0, \quad \dot{E}_{\text{in}} = \dot{E}_{\text{out}}, \quad \text{kJ/s} \]
  \[ \dot{u}_{\text{in}} - \dot{u}_{\text{out}} + \dot{W}_{\text{i}} - \dot{W}_{\text{out}} = \left( \dot{m} \frac{\partial u}{\partial \theta} \right)_{\text{in}} - \left( \dot{m} \frac{\partial u}{\partial \theta} \right)_{\text{out}}, \quad \text{kW} \]

\[ \dot{u}_{\text{in}} - \dot{u}_{\text{out}} = \theta_{\text{out}} - \theta_{\text{in}}, \quad \text{kJ/kg} \]

- **Energy balance**
  \[ \dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \dot{W}_{\text{i}} - \dot{W}_{\text{out}}, \quad \text{kW} \]

**Second Law**

**Thermal Efficiency for steam power plants**

\[ \eta = \frac{\text{desired output}}{\text{required input}} = \frac{\dot{u}_{\text{net, out}}}{q_{\text{in}}} \]

\[ \eta = \frac{\dot{u}_{\text{net, out}}}{q_{\text{in}}} = \frac{q_{\text{in}} - q_{\text{out}}}{q_{\text{in}}} = \frac{q_{\text{in}}}{q_{\text{in}}} - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{\dot{u}_{\text{out}}}{\dot{u}_{\text{in}}} \]

**Coefficient of Performance for a Refrigerator**

\[ \text{COP}_R = \frac{\text{desired output}}{\text{required input}} = \frac{q_{\text{in}}}{\dot{u}_{\text{net, in}}} \]

\[ \text{COP}_R = \frac{q_{\text{in}}}{\dot{u}_{\text{net, in}}} = \frac{q_{\text{out}} - q_{\text{in}}}{q_{\text{in}}} = \frac{q_{\text{out}}}{q_{\text{in}}} - 1 = 1 - \frac{q_{\text{in}}}{q_{\text{out}}} \]

\[ \text{COP}_R = \frac{q_{\text{in}}}{\dot{u}_{\text{net, in}}} = \frac{q_{\text{out}}}{q_{\text{in}}} - 1 = \frac{q_{\text{in}}}{q_{\text{in}}} - 1 = 1 \]

Second Law

**Purpose:** Produce work, \( W_{\text{out}} = \dot{u}_{\text{out}} \)

Second Law

**Purpose:** Maintain space at low T by removing \( q_L \)

Second Law

**Purpose:** Maintain space at high T by supplying \( q_H \)
Second Law

Coefficient of Performance for a Heat Pump

\[ \text{COP}_{HP} = \frac{\text{desired output}}{\text{required input}} = \frac{q_{out}}{w_{net,in}} \]

\[ \text{COP}_{sgp} = \frac{q_{out}}{w_{net,in}} \frac{q_{out}}{q_{out} - q_{in}} \]

\[ = \frac{1}{q_{out} - q_{in}} \frac{1}{1 - q_{in}/q_{in}} \]

Second Law – Energy Degrade

What is the maximum performance of real engines if it can never achieve 100%??

Factors of irreversibilities

- less heat can be converted to work
- Friction between 2 moving surfaces
- Processes happen too fast
- Non-isothermal heat transfer

Second Law – Dream Engine

Carnot Cycle

- **Isothermal expansion**
  - Slow adding of Q resulting in work done by system (system expand)
  - \( Q_e - W_{out} = \Delta U = 0 \), So, \( Q_e = W_{out} \). Pressure drops.

- **Adiabatic expansion**
  - \( 0 = W_{out} = \Delta U \). Final U smaller than initial U.
  - T & P drops.

Second Law – Dream Engine

Carnot Cycle

- **Isothermal compression**
  - Work done on the system
  - Slow rejection of Q
  - \( -Q_{out} + W_{in} = \Delta U = 0 \). So, \( Q_{out} = W_{in} \).
  - Pressure increases.

- **Adiabatic compression**
  - \( 0 + W_{in} = \Delta U \). Final U higher than initial U.
  - T & P increases.

Second Law – Dream Engine

Carnot Cycle

- P - \( v \) diagram for a Carnot (ideal) power plant

Second Law – Dream Engine

Reverse Carnot Cycle

- P - \( v \) diagram for a Carnot (ideal) refrigerator
Second Law – Dream Engine
Carnot Principles
• For heat engines in contact with the same hot and cold reservoir
  - All reversible engines have the same performance.
  - Real engines will have lower performance than the ideal engines.

\[
\left( \frac{q_H}{q_L} \right)_{rev} = \frac{T_H}{T_L} (K)
\]

Second Law – Will a Process Happen
Carnot Principles
• For heat engines in contact with the same hot and cold reservoir
  - Equality
  - Inequality

Processes satisfying Carnot Principles obey the Second Law of Thermodynamics

Clausius Inequality:
• Sum of $Q/T$ in a cyclic process must be zero for reversible processes and negative for real processes

\[
\int \frac{\delta Q}{T} \leq 0, \frac{kJ}{K}, \quad \int \frac{\delta q}{T} \leq 0, \frac{kJ}{kg \cdot K},
\]

\[
\int \frac{\delta Q}{T} = 0, \text{ reversible}
\]

\[
\int \frac{\delta Q}{T} > 0, \text{ impossible}
\]
Entropy – Quantifying Disorder

Entropy

- Quantifies lost of energy quality
- Can be transferred by heat and mass or generated due to irreversibility factors:
  - Frictional forces between moving surfaces.
  - Fast expansion & compression.
  - Heat transfer at finite temperature difference.

System

\[
\Delta S_{sys} = \int_{sys} \left( \frac{\delta Q}{T} \right)_{int\ rev} \quad \Delta S_{sys} = \frac{Q_{out}}{T_H} + \frac{Q_{in}}{T_L} + \frac{Q_{fric}}{T_L} + \frac{Q_{irr}}{T_L} 
\]

Sink

\[
\Delta S = S_2 - S_1 = \int \left( \frac{\delta Q}{T} \right)_{int\ rev} \quad \Delta S_{sys} = \Delta S_{in} = \frac{Q_{in}}{T_L} + \frac{Q_{fric}}{T_L} + \frac{Q_{irr}}{T_L} 
\]

Source

\[
\Delta S_{source} = Q_{out} \frac{1}{T_H} = \frac{Q_{in}}{T_H} + \frac{Q_{fric}}{T_H} + \frac{Q_{irr}}{T_H} 
\]

Steam Power Plant

\[
\delta S = \frac{\delta Q}{T} 
\]

Entropy Change in a process

\[
\Delta S = S_2 - S_1 = \int \left( \frac{\delta Q}{T} \right)_{int\ rev} 
\]

Increase of Entropy Principle

The entropy of an isolated (closed and adiabatic) system undergoing any process, will always increase.

\[
\Delta S_{isolated} = \Delta S_{sys} + \Delta S_{sur} \geq 0 
\]

For pure substance:

\[
\Delta S_{sys} = m(s_2 - s_1) 
\]

\[
\Delta S_{sur} = \frac{(Q_{in} - Q_{out})_{sur}}{T_{sur}} 
\]

Increase of Entropy Principle Proven

Consider the following cyclic process containing an irreversible forward path and a reversible return path

\[
\int \delta Q = \int \left( \frac{\delta Q}{T} \right)_{int\ rev} \leq 0 
\]

So:

\[
\int \frac{\delta Q}{T} + S_1 - S_2 \leq 0 
\]

Then:

\[
S_2 - S_1 \geq \int \left( \frac{\delta Q}{T} \right)_{irrev} 
\]

Entropy – Quantifying Disorder

Entropy

- Quantitative measure of disorder or chaos
- Is a system’s property, just like the others
- Does not depend on process path
- Has values at every state

Entropy

9 Quantifies lost of energy quality
9 Can be transferred by heat and mass or generated due to irreversibility factors:
- Frictional forces between moving surfaces.
- Fast expansion & compression.
- Heat transfer at finite temperature difference.

Entropy Change

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\Delta S_{sys} = m(s_2 - s_1) 
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\Delta S_{sur} = \frac{(Q_{in} - Q_{out})_{sur}}{T_{sur}} 
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Increase of Entropy Principle Proven

Consider the following cyclic process containing an irreversible forward path and a reversible return path

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So:

\[
\int \frac{\delta Q}{T} + S_1 - S_2 \leq 0 
\]

Then:

\[
S_2 - S_1 \geq \int \left( \frac{\delta Q}{T} \right)_{irrev} 
\]
Increase of Entropy Principle Proven

Consider the following cyclic process containing an irreversible forward path and a reversible return path.

Then entropy change for the closed system:

\[ \Delta S_{\text{sys}} = S_2 - S_1 = \int \frac{\delta Q}{T} + S_{\text{gen}} \]

For adiabatic system:

\[ \Delta S_{\text{iso sys}} = S_2 - S_1 = 0 + 0 \]

Isentropic or constant entropy process

For isolated (adiabatic & closed) system:

\[ S_{\text{gen}} = \Delta S_{\text{sys}} + \Delta S_{\text{sur}} \geq 0 \]

\[ q_{\text{in}} = q_{\text{H}} \]

Area under T-S diagram is amount of heat in a process

\[ dA = Tds = q_{\text{in}} \]

The infinite area \( dA \) is the area of a strip = \( Tds \)

\[ \Delta A = \int dA = \int Tds = q_{\text{in}} \]

Adding all the area of the strips from state 1 to state 2 will give the total area under process curve. It represents specific heat received for this process.
Factors affecting Entropy (disorder)

- Entropy will change when there is
  - Heat transfer (receiving heat increases entropy)
  - Mass transfer (moving mass changes entropy)
  - Irreversibilities (entropy will always be generated)

Entropy Balance

For any system undergoing any process,

- Energy must be conserved (\( E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{sys}} \))
- Mass must be conserved (\( m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{sys}} \))
- Entropy will always be generated except for reversible processes

Entropy balance is (\( S_{\text{in}} - S_{\text{out}} + S_{\text{gen}} = \Delta S_{\text{sys}} \))

Entropy Balance – Closed system

Energy Balance: \( q_{\text{in}} - q_{\text{out}} + \omega_{\text{in}} - \omega_{\text{out}} = \Delta u + \Delta ke + \Delta pe \)

Entropy Balance: \( s_{\text{in}} - s_{\text{out}} + s_{\text{gen}} = \Delta s_{\text{sys}}, \frac{kJ}{kg} \cdot K \)

\( (s_{\text{heat}} + s_{\text{mass}})_{\text{in}} - (s_{\text{heat}} + s_{\text{mass}})_{\text{out}} + s_{\text{gen}} = \Delta s_{\text{sys}}, \frac{kJ}{kg} \cdot K \)

\( s_{\text{gen}} = \Delta s_{\text{sys}} + (s_{\text{heat}} + 0)_{\text{out}} - (s_{\text{heat}} + 0)_{\text{in}}, \frac{kJ}{kg} \cdot K \)

\( s_{\text{gen}} = m(s_{2} - s_{1})_{\text{sys}} + \left( \frac{q_{\text{out}} - q_{\text{in}}}{T_{\text{in}}/T_{\text{sys}}} \right) \cdot \frac{kJ}{kg} \cdot K \)

Entropy Balance – Steady-flow device

Nozzle:

Assume adiabatic, no work done, \( \Delta p_{\text{inlet}} = \Delta p_{\text{exit}} \), \( \Delta p_{\text{exit}} = 0 \)

Energy Balance:

\( q_{\text{in}} - q_{\text{out}} + \dot{W}_{\text{in}} - \dot{W}_{\text{out}} = \dot{m}(h_{2} - h_{1}) \), kW

Assume adiabatic, \( \Delta k_{\text{mass}} = 0 \), \( \Delta k_{\text{pe}} = 0 \)

Entropy Balance:

\( \dot{S}_{\text{gen}} = \frac{\dot{Q}_{\text{out}}}{T_{\text{out}}} - \frac{\dot{Q}_{\text{in}}}{T_{\text{in}}} + m_{2} s_{2} - m_{1} s_{1}, \frac{K}{W} \)

\( \dot{S}_{\text{gen}} = 0 - 0 + m(s_{2} - s_{1}), \frac{K}{W} \)

Turbine:

Assume adiabatic, \( \Delta k_{\text{mass}} = 0 \), \( \Delta k_{\text{pe}} = 0 \)

Energy Balance:

\( q_{\text{in}} - q_{\text{out}} + \dot{W}_{\text{in}} - \dot{W}_{\text{out}} = \dot{m}(h_{2} - h_{1}) \), kW

Assume adiabatic, \( \Delta k_{\text{mass}} = 0 \), \( \Delta k_{\text{pe}} = 0 \)

Entropy Balance:

\( \dot{S}_{\text{gen}} = \frac{\dot{Q}_{\text{out}}}{T_{\text{out}}} - \frac{\dot{Q}_{\text{in}}}{T_{\text{in}}} + m_{2} s_{2} - m_{1} s_{1}, \frac{K}{W} \)

\( \dot{S}_{\text{gen}} = 0 - 0 + m(s_{2} - s_{1}), \frac{K}{W} \)
Entropy Balance – Steady-flow device

**Entropy – Quantifying Disorder**

Heat exchanger: energy balance; 2 cases

where \( \dot{m}_{\text{inlet}} = \dot{m}_{\text{exit}} = \dot{m} \)

Assume \( \Delta k_{\text{mass}} = 0, \Delta p_{\text{mass}} = 0 \)

**Case 1**

\[
\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} + \dot{W}_{\text{in}} - \dot{W}_{\text{out}} = m(\dot{\vartheta}_{\text{out}} - \dot{\vartheta}_{\text{in}}) \text{ kW}
\]

\[
0 = \dot{m}_{1} h_{4} - \dot{m}_{1} h_{3} + \dot{m}_{2} h_{2} - \dot{m}_{1} h_{1}, \text{ kW}
\]

**Case 2**

\[
\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} = \dot{m}_{2} h_{2} - \dot{m}_{1} h_{1}, \text{ kW}
\]

\[
\dot{Q}_{\text{in}} - 0 = \dot{m}_{2} h_{2} - \dot{m}_{1} h_{1}, \text{ kW}
\]

**Case 1**

\[
\dot{S}_{\text{gen}} = 0 - 0 + \dot{m}_{4} s_{4} - m_{3} s_{3} + m_{2} s_{2} - \dot{m}_{1} s_{1}, \text{ kW/K}
\]

**Case 2**

\[
\dot{S}_{\text{gen}} = \frac{\dot{Q}_{\text{out}}}{T_{\text{out}}} - \frac{\dot{Q}_{\text{in}}}{T_{\text{in}}} + \frac{\dot{m}_{2} s_{2} - \dot{m}_{1} s_{1}}{K}, \text{ kW/K}
\]

Mixing Chamber:

\[
\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} + \dot{W}_{\text{in}} - \dot{W}_{\text{out}} = \sum \dot{m} \vartheta_{\text{exit}} - \sum \dot{m} \vartheta_{\text{inlet}}, \text{ kW}
\]

\[
\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} + \dot{W}_{\text{in}} - \dot{W}_{\text{out}} = \dot{m}_{1} h_{4} - \dot{m}_{2} h_{2} - \dot{m}_{1} h_{1}, \text{ kW}
\]

\[
\dot{S}_{\text{gen}} = \frac{\dot{Q}_{\text{out}}}{T_{\text{out}}} - \frac{\dot{Q}_{\text{in}}}{T_{\text{in}}} + \frac{\dot{m}_{2} s_{2} - \dot{m}_{1} s_{1}}{K}, \text{ kW/K}
\]

where \( \dot{m}_{\text{inlet}} = \dot{m}_{\text{exit}} = \dot{m} \)