NAME: KEY

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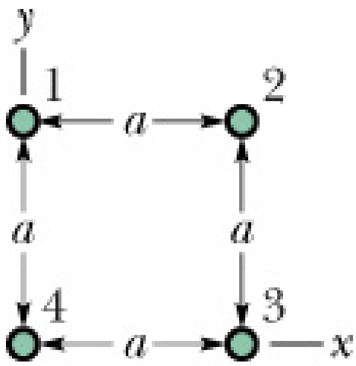
TEST 3 –– October 19th 2006

Answer ALL questions ON the paper provided to you. DO NOT USE ADDITIONAL PAPERS.



QUESTION 1

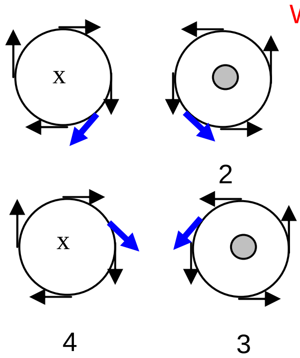
Four long straight wires are perpendicular to the page of the paper and separated by distances ***a*** as shown in the diagram. Currents in wire **1 and wire 4 are <u>into the page</u>** and currents in wire **2 and 3 are <u>out of the page</u>**.

a) Write the magnitude of the magnetic fields B_1, B_2, B_3 and B_4, respectively, produced by each of the wires **at the center of the square**. Then indicate the directions of each B field using the normal convention as shown.

(10 marks)

<u>Solution</u>: (2 marks for the Pythagoras thrm, 1 each for the field & 1 for the direction. 2+8=10)

The figure below shows the B field produced by each wire



Directions of B field surrounding each wire. The blue color show directions at the center of the square

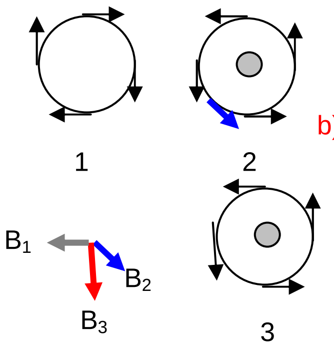
Wire 1: B points southwest. The distance from the wires to the center of the square is obtained by using Pythagoras theorem:

$r^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 = \frac{2a^2}{4} = \frac{a^2}{2}$. Then $B_1 = \frac{\mu_0 I_1}{2\pi \frac{a}{\sqrt{2}}}$

Wire 2: B field points southeast. $B_2 = \frac{\mu_0 I_2}{2\pi \frac{a}{\sqrt{2}}}$

Wire 3: B field points southwest. $B_3 = \frac{\mu_0 I_3}{2\pi \frac{a}{\sqrt{2}}}$

Wire 4: B field points southeast. $B_4 = \frac{\mu_0 I_4}{2\pi \frac{a}{\sqrt{2}}}$



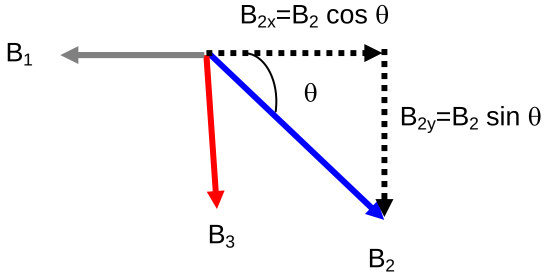
Directions of B field surrounding each wire at the position of wire 4.

b) (i) Wire 1: B points west. Then $B_1 = \frac{\mu_0 I_1}{2\pi a}$

Wire 2: B points southwest. Then $B_2 = \frac{\mu_0 I_2}{2\pi a\sqrt{2}}$

Wire 3: B points south Then $B_3 = \frac{\mu_0 I_3}{2\pi a}$

(ii) Obtain the net magnetic field **at the position of wire 4.**



The total B field is obtained by adding the x and y components of all the three fields.

$$B_x = B_{2x} - B_1 = \frac{\mu_0 I_2 \cos\theta}{2\pi a\sqrt{2}} - \frac{\mu_0 I_1}{2\pi a}$$

$$B_y = -B_{2y} - B_3 = -\frac{\mu_0 I_2 \sin\theta}{2\pi a\sqrt{2}} - \frac{\mu_0 I_3}{2\pi a}.$$

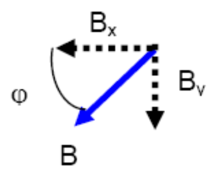
Then,

$$B = \sqrt{B_x^{\ 2} + B_y^{\ 2}} = \sqrt{\left(\frac{\mu_0 I_2 \cos\theta}{2\pi a\sqrt{2}} - \frac{\mu_0 I_3}{2\pi a}\right)^2 + \left(-\frac{\mu_0 I_2 \sin\theta}{2\pi a\sqrt{2}} - \frac{\mu_0 I_3}{2\pi a}\right)^2}$$

$$\tan\phi = \frac{B_y}{B_x} = \frac{\left(-\frac{\mu_0 I_2 \sin\theta}{2\pi a\sqrt{2}} - \frac{\mu_0 I_3}{2\pi a}\right)}{\left(\frac{\mu_0 I_2 \cos\theta}{2\pi a\sqrt{2}} - \frac{\mu_0 I_1}{2\pi a}\right)}.$$

If we assume the currents are all the same, $I_1 = I_2 = I_3 = I_4 = I$ and that the angle theta is 45 degrees from the symmetry of the problem, then the solution can easily be solved since $\sin\theta = \cos\theta = \frac{1}{\sqrt{2}}$. (use Pythagoras theorem), Then

$$B_x = \frac{\mu_0 I}{2\pi a\sqrt{2}\sqrt{2}} - \frac{\mu_0 I}{2\pi a} = \frac{\mu_0 I}{2\pi a}\left(\frac{1}{2} - 1\right) = -\frac{\mu_0 I}{4\pi a}$$ and



$$B_y = -\frac{\mu_0 I}{2\pi a\sqrt{2}\sqrt{2}} - \frac{\mu_0 I}{2\pi a} = \frac{\mu_0 I}{2\pi a}\left(-\frac{1}{2} - 1\right) = -\frac{3\mu_0 I}{4\pi a}.$$ Then

$$B = \sqrt{\left(-\frac{\mu_0 I}{4\pi a}\right)^2 + \left(-\frac{3\mu_0 I}{4\pi a}\right)^2} = \frac{\mu_0 I}{4\pi a}\sqrt{10}$$

$$\tan\phi = \frac{B_y}{B_x} = \left\| \frac{\left(-\frac{3\mu_0 I}{4\pi a}\right)}{\left(\frac{\mu_0 I}{4\pi a}\right)} \right\| = 3.$$ So $\phi = \tan^{-1}(3). = 71.56^\circ$

(iii) Obtain the force per unit length, $\frac{F}{l}$, **acting on wire 4.**

Since the force is $F_B = IL_4 B_{123} \sin\theta_{IB}$ and that the B field is in the plane of the paper which is perpendicular to the current I_4, then $\frac{F_B}{L_4} = IB_{123} = \frac{\mu_0 I^2}{4\pi a}\sqrt{10}$

(6+11+3=20 marks)

QUESTION 2

(a) Inductance of an inductor is just the amount of magnetic flux linkage for every ampere of current through the coils of the inductor and can be written mathematically as $L = \frac{N\Phi_B}{I}$. Show that the inductance per unit length near the center of a long solenoid is $\frac{L}{l} = \mu_0 n^2 A$ where n is the number of turns per unit length of the wire and A is the cross sectional area of the solenoid.

Solution:

For a short solenoid, the B field near the center is $B = \mu_0 nI$ where n = number of turns for each meter or simply, $n = N/\ell$. Since the flux linkage which gave rise to the inductance is $\Phi_B = BA$ where A is the surface area which is parallel to the B field, than the inductance is

$$L = \frac{N\Phi_B}{I} = \frac{NBA}{I} = \frac{N\mu_0 nIA}{I} = n\ell\mu_0 nA = \mu_0 n^2 \ell A.$$ Then $\frac{L}{\ell} = \mu_0 n^2 A$.

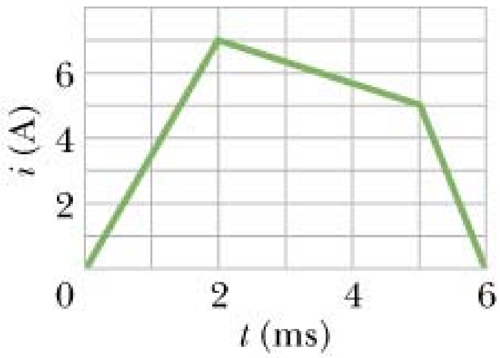
(5 marks)

(b)

The current *i* through a 4.0 H inductor is changing with time as shown in the graph. Find the magnitude of the induced emf ξ, during the intervals;

(i) 0 to 2 seconds
(ii) 2 to 5 seconds, and
(iii) 5 to 6 seconds.

Explain the results you obtain.



Solution:

Since the emf induced is proportional to the flux change and the flux is directly proportional to the current change, than we just need to find the rate of current change.

(i) $$\varepsilon = -L\frac{\Delta i}{\Delta t} = -(4mH)\frac{6-2}{2-0} = -8\ mV$$

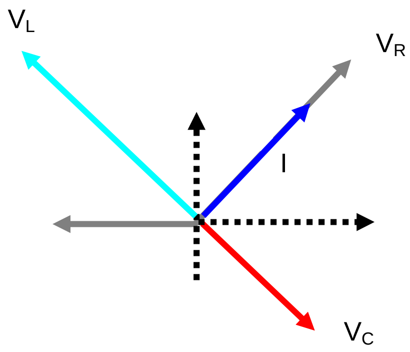
(ii) $$\varepsilon = -L\frac{\Delta i}{\Delta t} = -(4mH)\frac{4-6}{4-2} = 4\ mV$$

(iii) $$\varepsilon = -L\frac{\Delta i}{\Delta t} = -(4mH)\frac{0-4}{6-4} = 8mV$$

Positive emf induced is because the current is decreasing while negative emf induced is because the current is increasing

(8 marks)

(c) Sketch a PHASOR diagram and label all the quantities when a circuit containing a resistor, an inductor and a capacitor is connected in series to an AC source. Then write down the reactances, X_L, and X_C, in terms of the frequency f. Explain what happens to the reactances for extremely high f and extremely low f.



Solution: The phasors are shown in the figure. The reactances are:

Inductor: $X_L = \omega L = 2\pi f L$. For very high f, X_L approaches infinity and acts as an insulator to the AC source. For very low f, X_L approaches zero and the inductor behaves like a short circuit

Capacitor: $X_C = \dfrac{1}{\omega L} = \dfrac{1}{2\pi f C}$. For very high f, X_C approaches zero and the inductor behaves like a short circuit. For very low f, X_C approaches infinity and acts as an insulator to the AC source.

(12 marks)

(d) State the meaning of the following concepts: *Lagging, leading, resonance, refraction, diffraction and interference*

Lagging usually refers to the voltage falling behind or being out of phase from the current by +90 degrees.
Leading usually refers to the voltage being in front or out of phase from the current by -90 degrees.
Resonance is a phenomena where the current in an RLC circuit is the maximum and it can only happen when $X_L = X_C$, and the impedance is then only attributed to the resistor in the circuit.
Refraction is the bending of light upon entering a medium with different index of refraction such as from air to glass.
Diffraction is the ability of a wave to bend around an obstacle or around the edges of an opening.
Interference is a phenomena of bright and dark fringes being observed on a screen due to the effect of superposition (adding constructively or destructively) of waves

(10 marks)

(e) The wavelength of yellow sodium light in air is 589 nm. Using the speed of light as $3x10^8$ *m/s*,

(i) determine its frequency,

Since $c = f\lambda$, then the frequency is: $f = \dfrac{c}{\lambda} = \dfrac{3\times10^8\ m/s}{589\ nm} = \dfrac{3000\times10^{5+9}\ m/s}{589\ m} = 5.093\times10^{14}\ Hz$

(ii) Noting the fact that the index of refraction of a medium is, $n = \dfrac{c}{v_n} = \dfrac{\lambda}{\lambda_n}$, where v_n is the speed of light in the medium. Obtain the wavelength of yellow sodium light, λ_n, upon entering glass whose index of refraction is 1.52.

Since $n = \dfrac{\lambda}{\lambda_n}$, then $\lambda_n = \dfrac{\lambda}{n} = \dfrac{589\ nm}{1.52} = 387.5\ nm$

(iii) Will the frequency and the speed of light change upon entering glass? Why or why not? Proof your claim.

The frequency remains the same. When light enters a denser medium, the path length of the wave front gets closely spaced, hence wavelength becomes shorter. Since the frequency remains the same, hence the speed must also become smaller. The proof can be shown as follows:

$\frac{v_n}{c} = \frac{f\lambda_n}{f\lambda} = \frac{\lambda_n}{\lambda} < 1$. Since the wavelength is smaller in material medium hence the speed will be smaller than 3×10^8 m/s.

(12 marks)