Electric Charges

Objectives/Learning outcome:
1. Explain the concept of electric field around a point charge and electric forces experienced by charged particles moving in electric fields.
2. Write mathematical relations between electric field, electric forces and acceleration of charged particles moving in electric fields.
3. Describe and draw the trajectory of charged particles moving in an electric field.
4. Determine final vertical velocity, final acceleration and final vertical position by applying Newton’s laws.

Electric Charges

Objectives/Learning outcome:
5. Draw and explain magnetic fields and magnetic forces.
6. Write mathematical relations for magnetic forces acting on charged particles moving in magnetic fields.
7. Use Right-Hand-Rule 1 to determine direction of forces exerted on charged particles moving in magnetic fields.
8. Explain the trajectory of charged particles moving in magnetic fields and obtain the radius of trajectory.

Electric Charges

Objectives/Learning outcome:
9. Describe the working of a velocity selector by drawing and explaining the trajectory of charged particles entering a magnetic/electric field.
10. Describe the working of a mass spectrometer.
11. Explain the working mechanism of cathode ray tubes.
12. Use mathematical relations to solve problems related to velocity selectors and mass spectrometers.

Electric Point Charges

Electric forces: attractive forces since opposite charges
\[ F_{12} = \frac{k \times q_1 \times q_2}{r^2} \]
\[ F_{12} = \frac{2 \times 2 \times e \times e}{r^2} = \frac{4 \times e^2}{r^2} \]
Is \( F_{net1} \) bigger or smaller than \( F_{net2} \)?

Electric Field

Electric Field: an area where any charged object will experience an electric force
\[ E = \frac{F}{q} = k \times \frac{q}{r^2} \]
Electric Field

Electric Field: Place the point charge $q$, at the center of an imaginary spherical surface of radius $r$ (gaussian surface).

The electric field strength is then directly proportional to the surface charge density, amount of charges in 1 square meter.

$$E = \frac{q}{4 \pi r^2 \varepsilon_0}$$

But the surface area of a sphere is $4\pi r^2$

$$E = \frac{q}{\varepsilon_0 A} = \frac{q}{\varepsilon_0}$$

Capacitors

Parallel plate capacitors: Two metal (conducting) plates, parallel to each other and separated by non-conductors such as air, paper, plastic etc. Use to store charges and hence store electrical energy.

The strength of the E field from each plate depends on the total charge within the Gaussian surface.

$$E = \frac{q}{\varepsilon_0 A}$$

Let the surface charge density or total charge per square meter be, $\sigma$.

E fields-capacitors

Parallel plate capacitors: Positive test charges placed between plates will experience electric forces. Hence, it will accelerate, consistent with Newton’s 2nd Law.

Charges moving at a constant velocity will experience a force only when entering an E field, hence it will accelerate.

E fields-capacitors

Parallel plate capacitors: Positive charges moving horizontally at velocity $v$, will accelerate vertically down, consistent with Newton’s 2nd Law, due to the electric force in the downward direction.
E fields-capacitors

Parallel plate capacitors: Positive charges moving horizontally at velocity $v = v_0$, will accelerate vertically down, consistent with Newton’s 2nd Law, due to the electric force in the downward direction.

- Along x: No force along x. Then, $v_x$ same as $v_0$, where $v_x = (s_y - s_i) / \Delta t = s_y / \Delta t$. The time to travel across the length of the capacitor is $\Delta t = s_y / v_y$.

- Along y: Electric field pointing down, hence electric force pushing the positive charge down as long as it is in the E field. Then, charge accelerates downward. $v_y = 0$, $v_i = v_j$.

\[
\begin{align*}
a_y &= \frac{v_j - v_i}{\Delta t} = \frac{v_y}{\Delta t} = a_i \\
\therefore v_y &= a_i \Delta t
\end{align*}
\]

The vertical displacement is found by using linear kinematics with constant acceleration in the vertical direction:

\[
\begin{align*}
s_y &= v_i \Delta t + \frac{a_i \Delta t^2}{2} \\
s_y &= \frac{qV_s}{md} \Delta t^2 = \frac{qV}{md} s_y
\end{align*}
\]

Magnetic Fields and Magnetic Forces

Magnetic Field

Magnetic Monopole: Electric charge can be separated until the smallest charge, the electron or proton. Magnets have isolated north and south pole and cannot be isolated into north only or south only.
Magnetic Field

Magnetic Monopole: Electric charge can be separated until the smallest charge, the electron or proton. Magnets have isolated north and south pole and cannot be isolated into north only or south only. 

Like poles repel and unlike (opposite) poles attract.

Needle of compass is labeled with north seeking and south seeking poles.

Earth as a Magnet: The earth magnetic north is where the magnetic axis crosses the surface in the northern hemisphere. Its geographic north is where earth’s axis of rotation crosses the surface in the northern hemisphere.

Field lines: Magnetic field lines produced by a bar magnet points from the north to the south as shown below.

Compasses placed near the bar magnet shows the field directions.

Field lines: Magnetic field lines produced by a bar magnet and u-shaped magnet as shown below.

Field lines for permanent bar magnet and The strength of B is bigger near the poles as seen by the lines which are close together. For horse-shoe magnet, the B field between poles are equally strong.

Field lines: Magnetic field is a region where moving charges may experience a magnetic force.

The charge is moving in the same direction or opposite to B. Hence charge don’t feel magnetic force. The charge is moving perpendicular to B. Hence maximum force is experienced.
Magnetic Field

Field lines: Magnetic field is a region where moving charges may experience a magnetic force.

\[ \vec{F} = q(\sin \theta) \vec{B} \]

The charge is moving at an angle \( \theta \) with \( \vec{B} \). Only the perpendicular component of the velocity causes it to experience force.

The velocity is the sum of the perpendicular and parallel components:

\[ v_{\text{per}} = v \sin(\theta) \]
\[ v_{\text{par}} = v \cos(\theta) \]

When \( v \) and \( B \) are parallel or anti-parallel, then the angle \( \theta = 0 \), hence \( F = 0 \).

When \( v \) and \( B \) are perpendicular, then the angle \( \theta = 90^\circ \), hence \( F \neq 0 \), \( F = F_{\text{max}} \).

Direction of the resultant force is determined by using first right hand rule:

- Point all fingers in \( \vec{B} \) direction.
- Thumb in \( v \) direction, then palm will face direction of force.

\[ \vec{F} = q \vec{v} \times \vec{B} = q v \sin(\theta) \vec{B} \]

Note the following vector representation:

- Out of page
- Into page

Determine the direction of the resultant force for the following cases:

i) \( v \) and \( B \) into page
ii) \( v \) into page, \( B \) out of page
iii) \( v \) out of page, \( B \) to the right
iv) \( B \) to the left, \( v \) out of page, then the force points upward. Then charge accelerates upward.
Magnetic Force

**Magnetic force**: Positive charges moving horizontally from the left at velocity \( v \), will accelerate vertically down, consistent with Newton’s 2nd Law, due to the magnetic force in the downward direction. If it stays in the field, then its path will be a circle.

\[
F_B = q(v \sin \theta)B
\]

\[
F_B = q(v \sin 90^\circ)B
\]

\[
F_B = qvB
\]

Example: A proton moving in a B field

\[
\theta = \sin^{-1}(\theta)
\]

\[
BvqF vBB
\]

\[
\theta = \sin^{-1}(\theta)
\]

\[
BvqFB
\]

\[
qvB
\]

For circular motion, the force is

\[
F_B = \frac{mv^2}{r} = qvB
\]

Example: A proton moving at \( v = 5.0 \times 10^6 \) m/s, enters a magnetic field of 0.40 T at an angle of 30°. Determine (a) the magnetic force and the acceleration of the proton. (b) repeat for an electron.

For proton, charge is positive. \( q = 1.6 \times 10^{-19} \) C.

\[
v_{per} = v\sin \theta
\]

\[
v_{par} = v\cos \theta
\]

\[
BvqFB
\]

\[
qvB
\]

Using Newton’s laws and \( m_p = 1.67 \times 10^{-27} \) kg. and \( m_e = 9.11 \times 10^{-31} \) kg

\[
F = ma
\]

\[
\alpha = \frac{F}{m}
\]

Hence, \( \alpha = 9.6 \times 10^{13} \) m/s

\[
\alpha = 1.8 \times 10^{17} \) m/s

Magnetic-Electric Forces

**Parallel plate capacitors**: Positive charges moving at velocity \( v \), entering an E field is deflected sideways

**Magnetic force**: Positive charges entering the B field experience a deflection upwards. If it stays in the field, then its path will be a circle.

\[
F_B = qvB
\]

Charge In Crossed Electric and Magnetic Fields: Velocity Selector: Adjust E and B field to balance the forces: \( F_E = F_B \)

\[
\frac{qE}{qB} \frac{v}{B} = \frac{V}{dB}
\]

By adjusting the potential \( V \) between the plates, only the charges with speed \( v \) will be selected.
**Magnetic Force**

Example: A proton moving enters a 2.0 Tesla B-field and travels in a circle with \( r = 25 \text{ cm} \). Find its momentum and the frequency and period of the particle.

The charge moves in a circle and the force responsible for the trajectory is the magnetic force:

\[
F_{\text{centr}} = F_B = Bqv \sin 90^\circ = Bqv
\]

Then:

\[
\frac{mv^2}{r} = qv \sin 90^\circ B
\]

Momentum:

\[
mv = rqB
\]

Frequency \( f \), is the number of oscillations or revolution in 1 second. Period, \( T \), is time for 1 complete oscillation. So \( f = \frac{1}{T} \).

\[
B \times qv = \frac{2\pi}{T}
\]

\[
T = \frac{2\pi qv}{B}
\]

\[
f = \frac{1}{T} = \frac{B}{2\pi qv}
\]

**Mass Spectrometer**

A mass spectrometer is a device used to determine the relative masses and the abundance of isotopes of an element and identify unknown molecules produced in chemical reactions.

Atoms or molecules are vaporized and ionized (removal of electrons). The positive charges are then accelerated in a velocity selector before entering a B field. Only the chosen ions will strike the detector.

Using energy conservation:

\[
v_y^2 = 2(qV_x - V_y) l
\]

The radius is:

\[
r = \frac{mv}{qB}
\]

\[
F_C = F_B = \frac{mv^2}{r} = evB
\]

**Magnetic-Electric Forces**

Example: A proton is accelerated by the potential difference of 2100 V across the plates and entered a 0.01 T B-field which points out of page. Find exit velocity through the slit and determine the radius of the trajectory.

Since plate separation, \( d \), is not known, then use mechanical energy conservation. Total mechanical E is conserved.

\[
(KE + PE)_{\text{before}} = (KE + PE)_{\text{after}}
\]

Before: \( v_x = 0 \text{ m/s}, KE = \frac{1}{2}mv_x^2 = 0 \text{ kJ} \).

After: \( v_x = v_y/2 \text{ m/s}, EPE = \frac{1}{2}qV_x \)

Then, \( 0 + q(V_x - V_y) = \frac{1}{2}mv_y^2 \)

Hence, \( v_y^2 = 2q(V_x - V_y) l \)

\( v_y = 6.3 \times 10^{-5} \text{ m/s} \)

**Magnetic-Electric Forces**

Example: A proton is accelerated by the potential difference of 2100 V across the plates and entered a 0.01 T B-field which points out of page. Find exit velocity through the slit and determine the radius of the trajectory.

The proton leaves the electric field and enters the magnetic field. Since \( v \) is perpendicular to \( B \), the force \( F_x \) acts on the proton and pushes it to the right and causes a circular path to be traced.

\[
F_C = F_B = \frac{mv^2}{r} = qvB
\]

Then, \( r = 6.6 \times 10^{-2} \text{ m} \)

The radius is:

\[
r = \frac{mv}{qB}
\]

\[
F_C = \frac{mv^2}{r} = evB
\]
Magnetic-Electric Forces

Mass Spectrometer: Mass spectrum of naturally occurring neon with 3 isotopes.

\[ r = \left( \frac{qv}{2N} \right) B^2 \]