# Samples of solutions to conceptual problems from chapter 19 Cutnell & Johnson 7E

2. A positive point charge and a negative point charge have equal magnitudes. One charge is fixed to one corner of a square, and the other is fixed to another corner. On which corners should the charges be placed, so that the same potential exists at the empty corners? Give your reasoning.

2. *REASONING AND SOLUTION* The potential at a point in space that is a distance r from a point charge q is given by Equation 19.6: $V = kq / r$. When more than one point charge is present, the total potential at any location is the algebraic sum of the individual potentials created by each charge at that location.

A positive point charge and a negative point charge have equal magnitudes. One of the charges is fixed to one corner of a square. If the other charge is placed opposite to the first charge along the diagonal of the square, then each charge will be the same distance L from the empty corners. The potential at each of the empty corners will be

$$V = \frac{k(+Q)}{L} + \frac{k(-Q)}{L} = 0$$
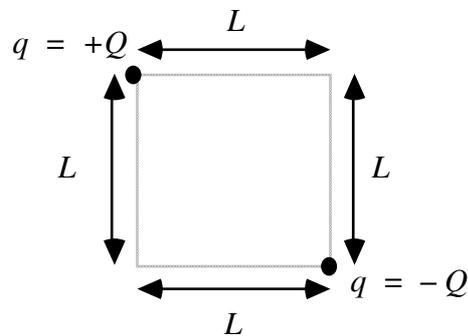
Therefore, if the potential at each empty corner is to be the same, then the charges must be placed at diagonally opposite corners as shown in the figure.

4. What point charges, all having the same magnitude, would you place at the corners of a square (one charge per corner), so that both the electric field and the electric potential (assuming a zero reference value at infinity) are zero at the center of the square? Account for the fact that the charge distribution gives rise to *both* a zero field and a zero potential.

4. *REASONING AND SOLUTION* Four point charges of equal magnitude are placed at the corners of a square as shown in the figure at the right.

The electric field at the center of the square is the *vector sum* of the electric field at the center due to each of the charges individually. The potential at the center of the square is equal to the *algebraic sum* of the potentials at the center due to each of the charges individually.
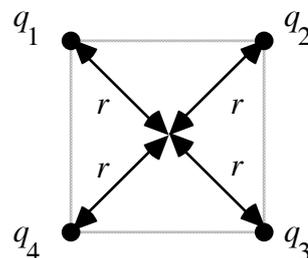
All four charges are equidistant from the center (a distance r in the figure). If two diagonal charges have the same magnitude and sign, then the electric field at the center due to these two charges have equal magnitude and opposite directions. Their resultant is, therefore, zero. Thus, the electric field at the center will be zero if each diagonal pair of charges has the same magnitude and sign.

If a diagonal pair of charges has the same magnitude and sign, they will give rise to a non-zero potential at the center. Thus, the potential due to one diagonal pair of charges must cancel the potential due to the other diagonal pair of charges. This will be the case if the two pairs of diagonal charges have opposite signs.

Thus, both the electric field and the electric potential will be zero at the center of the square if all four charges have the same magnitude, q_1 and q_3 have the same sign, and q_2 and q_4 have the same sign, which is opposite to the signs of q_1 and q_3.

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7. An electric potential energy exists when two protons are separated by a certain distance. Does the electric potential energy in crease, decrease, or remain the same when (a) both protons are replaced by electrons, and (b) only one of the protons is replaced by an electron? Justify your answers.

7. ***REASONING AND SOLUTION*** An electric potential energy exists when two protons are separated by a certain distance. It is equal to the work that must be done by an external agent to assemble the configuration. Suppose that we imagine assembling the system, one particle at a time. If there are no other charges in the region, there are no existing electric fields; therefore, no work is required to put the first proton in place. That proton, however, gives rise to an electric field that fills the region. Its magnitude at a distance r from the proton is given by Equation 18.3, $E = ke/r^2$, where $+e$ is the magnitude of the charge on the proton. Since the region contains an electric field due to the first proton, there also exists an electric potential, and the external agent must do work to place the second proton at a distance d from the first proton. The electric potential energy of the final configuration is equal to the work that must be done to bring the second proton from infinity and place it at a distance d from the first proton. The electric potential at a distance d from the first proton is $V_{\text{proton}} = +ke/d$ (Equation 19.6). According to Equation 19.3, the electric potential energy of the final configuration is therefore

$$\text{EPE} = V_{\text{proton}}(+e) = +\frac{ke^2}{d}$$

a. If both protons are replaced by electrons, similar arguments apply. However, since the electron carries a negative charge ($-e$), the electric potential at a distance d from the first electron is $V_{\text{electron}} = -ke/d$. The electric potential energy of the final configuration is now given by

$$\text{EPE} = V_{\text{electron}}(-e) = -\frac{ke}{d}(-e) = +\frac{ke^2}{d}$$

Therefore, if both protons are replaced by electrons, the electric potential energy *remains the same*.

b. When only one of the protons is replaced by an electron, we find that

$$\text{EPE} = V_{\text{proton}}(-e) = \left(+\frac{ke}{d}\right)(-e) = -\frac{ke^2}{d}$$

Thus, when only one of the protons is replaced by an electron, the electric potential energy *decreases* from $+ke^2/d$ to $-ke^2/d$.

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16. A proton and an electron are released from rest at the midpoint between the plates of a charged parallel plate capacitor. Except for these particles, nothing else is between the plates. Ignore the attraction between the proton and the electron, and decide which particle strikes a capacitor plate first. Why?

16. ***REASONING AND SOLUTION*** Since both particles are released from rest, their initial kinetic energies are zero. They both have electric potential energy by virtue of their respective positions in the electric field between the plates. Since the particles are oppositely charged, they move in opposite directions toward opposite plates of the capacitor. As they move toward the plates, the particles gain kinetic energy and lose potential energy. Using $(EPE)_0$ and $(EPE)_f$ to denote the initial and final electric potential energies of the particle, respectively, we find from energy conservation that

$$\left(EPE\right)_0 = \tfrac{1}{2} m_{particle} v_f^2 + \left(EPE\right)_f$$

The final speed of each particle is given by

$$v_f = \sqrt{\frac{2\left[\left(EPE\right)_0 - \left(EPE\right)_f\right]}{m_{particle}}}$$

Since both particles travel through the same distance between the plates of the capacitor, the change in the electric potential energy is the same for both particles. Since the mass of the electron is smaller than the mass of the proton, the final speed of the electron will be greater than that of the proton. Therefore, the electron travels faster than the proton as the particles move toward the respective plates. The electron, therefore, strikes the capacitor plate first.

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CHAPTER 19 | *Electric Potential Energy And The Electric Potential*

Samples of solutions to Problems from chapter 19 Cutnell & Johnson 7E

4. A particle has a charge of $+1.5\,\mu C$ and moves from point *A* to point *B*, a distance of 0.20 m. The particle experiences a constant electric force, and its motion is along the line of action of the force. The difference between the particle's electric potential energy at *A* and *B* is $EPE_A - EPE_B = +9.0 \times 10^{-4}\,J$. (a) Find the magnitude and direction of the electric force that acts on the particle. (b) Find the magnitude and direction of the electric field that the particle experiences.

4. ***REASONING*** Equation 19.1 indicates that the work done by the electric force as the particle moves from point *A* to point *B* is $W_{AB} = \text{EPE}_A - \text{EPE}_B$. For motion through a distance *s* along the line of action of a constant force of magnitude *F*, the work is given by Equation 6.1 as either $+Fs$ (if the force and the displacement have the same direction) or $-Fs$ (if the force and the displacement have opposite directions). Here, $\text{EPE}_A - \text{EPE}_B$ is given to be positive, so we can conclude that the work is $W_{AB} = +Fs$ and that the force points in the direction of the motion from point *A* to point *B*. The electric field is given by Equation 18.2 as $\mathbf{E} = \mathbf{F}/q_0$, where q_0 is the charge.

SOLUTION a. Using Equation 19.1 and the fact that $W_{AB} = +Fs$, we find

$$W_{AB} = +Fs = \text{EPE}_A - \text{EPE}_B$$

$$F = \frac{\text{EPE}_A - \text{EPE}_B}{s} = \frac{9.0 \times 10^{-4} \text{ J}}{0.20 \text{ m}} = \boxed{4.5 \times 10^{-3} \text{ N}}$$

As discussed in the reasoning, the direction of the force is $\boxed{\text{from } A \text{ toward } B}$.

b. From Equation 18.2, we find that the electric field has a magnitude of

$$E = \frac{F}{q_0} = \frac{4.5 \times 10^{-3} \text{ N}}{1.5 \times 10^{-6} \text{ C}} = \boxed{3.0 \times 10^3 \text{ N/C}}$$

The direction is the same as that of the force on the positive charge, namely $\boxed{\text{from } A \text{ toward } B}$.

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9. The potential at location *A* is 452 V. A positively charged particle is released there from rest and arrives at location *B* with a speed v_B. The potential at location *C* is 791 V, and when released from rest from this spot, the particle arrives at *B* with twice the speed it previously had, or $2v_B$. Find the potential at *B*

9. ***REASONING*** The only force acting on the moving charge is the conservative electric force. Therefore, the total energy of the charge remains constant. Applying the principle of conservation of energy between locations A and B, we obtain

$$\tfrac{1}{2}mv_A^2 + \text{EPE}_A = \tfrac{1}{2}mv_B^2 + \text{EPE}_B$$

Since the charged particle starts from rest, $v_A = 0$. The difference in potential energies is related to the difference in potentials by Equation 19.4, $\text{EPE}_B - \text{EPE}_A = q(V_B - V_A)$. Thus, we have

$$q(V_A - V_B) = \tfrac{1}{2}mv_B^2 \tag{1}$$

Similarly, applying the conservation of energy between locations C and B gives

$$q(V_C - V_B) = \tfrac{1}{2}m(2v_B)^2 \qquad\qquad (2)$$

Dividing Equation (1) by Equation (2) yields

$$\frac{V_A - V_B}{V_C - V_B} = \frac{1}{4}$$

This expression can be solved for V_B.

SOLUTION   Solving for V_B, we find that

$$V_B = \frac{4V_A - V_C}{3} = \frac{4(452 \text{ V})-791 \text{ V}}{3} = \boxed{339 \text{ V}}$$

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11. Two charges A and B are fixed in place, at different distances from a certain spot. At this spot the potentials due to the two charges are equal. Charge A is 0.18 m from the spot, while charge B is 0.43 m from it. Find the ratio q_B/q_A of the charges.

11. ***REASONING***   The potential of each charge q at a distance r away is given by Equation 19.6 as $V = kq/r$. By applying this expression to each charge, we will be able to find the desired ratio, because the distances are given for each charge.
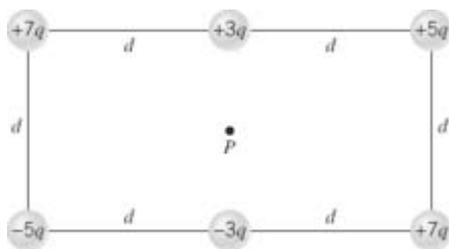
SOLUTION   According to Equation 19.6, the potentials of each charge are

$$V_A = \frac{kq_A}{r_A} \qquad \text{and} \qquad V_B = \frac{kq_B}{r_B}$$

Since we know that $V_A = V_B$, it follows that

$$\frac{kq_A}{r_A} = \frac{kq_B}{r_B} \qquad \text{or} \qquad \frac{q_B}{q_A} = \frac{r_B}{r_A} = \frac{0.43 \text{ m}}{0.18 \text{ m}} = \boxed{2.4}$$

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16. The drawing shows six point charges arranged in a rectangle. The value of q is $9.0\,\mu C$, and the distance d is 0.13 m. Find the total electric potential at location P, which is at the center of the rectangle.

16. **REASONING** The electric potential at a distance r from a point charge q is given by Equation 19.6 as $V = kq/r$. The total electric potential at location P due to the six point charges is the algebraic sum of the individual potentials.
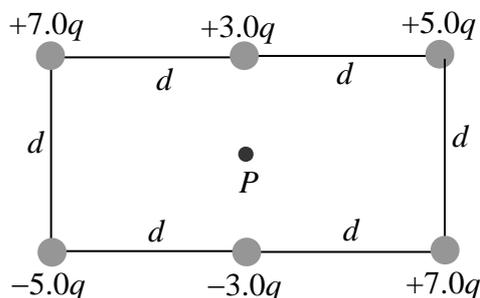
SOLUTION Starting at the upper left corner of the rectangle, we proceed clockwise and add up the six contributions to the total electric potential at P (see the drawing):

$$V = \frac{k(+7.0q)}{\sqrt{d^2 + \left(\dfrac{d}{2}\right)^2}} + \frac{k(+3.0q)}{\dfrac{d}{2}} + \frac{k(+5.0q)}{\sqrt{d^2 + \left(\dfrac{d}{2}\right)^2}} + \frac{k(+7.0q)}{\sqrt{d^2 + \left(\dfrac{d}{2}\right)^2}} + \frac{k(-3.0q)}{\dfrac{d}{2}} + \frac{k(-5.0q)}{\sqrt{d^2 + \left(\dfrac{d}{2}\right)^2}}$$

$$= \frac{k(+14.0q)}{\sqrt{d^2 + \left(\dfrac{d}{2}\right)^2}}$$

Substituting $q = 9.0 \times 10^{-6}$ C and $d = 0.13$ m gives

$$V = \frac{k(+14.0q)}{\sqrt{d^2 + \left(\dfrac{d}{2}\right)^2}} = \frac{\left(8.99 \times 10^9 \ \dfrac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)(+14.0)\left(9.0 \times 10^{-6} \text{ C}\right)}{\sqrt{(0.13 \text{ m})^2 + \left(\dfrac{0.13 \text{ m}}{2}\right)^2}} = \boxed{+7.8 \times 10^6 \text{ V}}$$

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37. The membrane that surrounds a certain type of living cell has a surface area of $5.0 \times 10^{-9} \text{ m}^2$ and a thickness of $1.0 \times 10^{-8} \text{ m}$. Assume that the membrane behaves like a parallel plate capacitor and has a dielectric constant of 5.0. (a) The potential on the outer surface of the membrane is +60.0 mV greater than that on the inside surface. How much charge resides on the outer surface? (b) If the charge in part (a) is due to K^+ ions (charge $+e$), how many such ions are present on the outer surface?

37. $\boxed{\text{SSM}}$ **REASONING** The charge that resides on the outer surface of the cell membrane is $q = CV$, according to Equation 19.8. Before we can use this expression, however, we must first determine the capacitance of the membrane. If we assume that the cell membrane behaves like a parallel plate capacitor filled with a dielectric, Equation 19.10 ($C = \kappa \varepsilon_0 A/d$) applies as well.

SOLUTION The capacitance of the cell membrane is

$$C = \frac{\kappa \varepsilon_0 A}{d} = \frac{(5.0)(8.85 \times 10^{-12}\ \text{F/m})(5.0 \times 10^{-9}\ \text{m}^2)}{1.0 \times 10^{-8}\ \text{m}} = 2.2 \times 10^{-11}\ \text{F}$$

a. The charge on the outer surface of the membrane is, therefore,

$$q = CV = (2.2 \times 10^{-11}\ \text{F})(60.0 \times 10^{-3}\ \text{V}) = \boxed{1.3 \times 10^{-12}\ \text{C}}$$

b. If the charge in part (a) is due to K^+ ions with charge $+e$ ($e = 1.6 \times 10^{-19}$ C), the number of ions present on the outer surface of the membrane is

$$\text{Number of } K^+ \text{ ions} = \frac{1.3 \times 10^{-12}\ \text{C}}{1.6 \times 10^{-19}\ \text{C}} = \boxed{8.1 \times 10^6}$$

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42. Two capacitors are identical, except that one is empty and the other is filled with a dielectric (k = 4.50). The empty capacitor is connected to a 12.0-V battery. What must be the potential difference across the plates of the capacitor filled with a dielectric such that it stores the same amount of electrical energy as the empty capacitor?

42. ***REASONING*** The energy used to charge up a capacitor is stored in the capacitor as electrical energy. The energy stored depends on the capacitance C of the capacitor and the potential difference V between its plates; $\text{Energy} = \frac{1}{2} CV^2$ (Equation 19.11b). Inserting a dielectric between the plates of a capacitor increases its capacitance by a factor of κ, where κ is the dielectric constant of the material. We will use these two pieces of information to find the potential difference across the plates of the capacitor filled with the dielectric.

SOLUTION The energy stored in the empty capacitor is $\text{Energy} = \frac{1}{2} C_0 V_0^2$, where C_0 is its capacitance and V_0 is the potential difference between its plates. Similarly, the energy stored in the capacitor filled with the dielectric is $\text{Energy} = \frac{1}{2} CV^2$, where C is its capacitance and V is the potential difference between its plates. Since the two energies are equal,

$$\tfrac{1}{2} C_0 V_0^2 = \tfrac{1}{2} CV^2$$

Since $C = \kappa C_0$ (see Equation 19.10 and the discussion that follows), we have

$$\tfrac{1}{2} C_0 V_0^2 = \tfrac{1}{2}\left(\kappa C_0\right) V^2$$

Solving for the potential difference V, gives

$$V = \frac{V_0}{\sqrt{\kappa}} = \frac{12.0\ \text{V}}{\sqrt{4.50}} = \boxed{5.66\ \text{V}}$$

59. The potential difference between the plates of a capacitor is 175 V. Midway between the plates, a proton and an electron are released. The electron is released from rest. The proton is projected perpendicularly toward the negative plate with an initial speed. The proton strikes the negative plate at the same instant that the electron strikes the positive plate. Ignore the attraction between the two particles, and find the initial speed of the proton.

59. *REASONING* If we assume that the motion of the proton and the electron is horizontal in the $+x$ direction, the motion of the proton is determined by Equation 2.8, $x = v_0 t + \frac{1}{2} a_{\mathrm{p}} t^2$, where x is the distance traveled by the proton, v_0 is its initial speed, and a_{p} is its acceleration. If the distance between the capacitor places is d, then this relation becomes $\frac{1}{2} d = v_0 t + \frac{1}{2} a_{\mathrm{p}} t^2$, or

$$d = 2v_0 t + a_{\mathrm{p}} t^2 \tag{1}$$

We can solve Equation (1) for the initial speed v_0 of the proton, but, first, we must determine the time t and the acceleration a_{p} of the proton . Since the proton strikes the negative plate at the same instant the electron strikes the positive plate, we can use the motion of the electron to determine the time t.

For the electron, $\frac{1}{2} d = \frac{1}{2} a_{\mathrm{e}} t^2$, where we have taken into account the fact that the electron is released from rest. Solving this expression for t we have $t = \sqrt{d / a_{\mathrm{e}}}$. Substituting this expression into Equation (1), we have

$$d = 2v_0 \sqrt{\frac{d}{a_e}} + \left(\frac{a_{\mathrm{p}}}{a_{\mathrm{e}}} \right) d \tag{2}$$

The accelerations can be found by noting that the magnitudes of the forces on the electron and proton are equal, since these particles have the same magnitude of charge. The force on the electron is $F = eE = eV / d$, and the acceleration of the electron is, therefore,

$$a_{\mathrm{e}} = \frac{F}{m_{\mathrm{e}}} = \frac{eV}{m_{\mathrm{e}} d} \tag{3}$$

Newton's second law requires that $m_{\mathrm{e}} a_{\mathrm{e}} = m_{\mathrm{p}} a_{\mathrm{p}}$, so that

$$\frac{a_{\mathrm{p}}}{a_{\mathrm{e}}} = \frac{m_{\mathrm{e}}}{m_{\mathrm{p}}} \tag{4}$$

Combining Equations (2), (3) and (4) leads to the following expression for v_0, the initial speed of the proton:

$$v_0 = \frac{1}{2}\left(1 - \frac{m_e}{m_p}\right)\sqrt{\frac{eV}{m_e}}$$

SOLUTION Substituting values into the expression above, we find

$$v_0 = \frac{1}{2}\left(1 - \frac{9.11\times10^{-31}\,\text{kg}}{1.67\times10^{-27}\,\text{kg}}\right)\sqrt{\frac{(1.60\times10^{-19}\,\text{C})(175\text{ V})}{9.11\times10^{-31}\,\text{kg}}} = \boxed{2.77\times10^6\,\text{m/s}}$$