Samples of solutions to conceptual problems from chapter 18 Cutnell & Johnson 7E

2. A metallic object is given a positive charge by the process of induction, as illustrated in Figure 18.8. (a) Does the mass of the object increase, decrease, or remain the same? Why? (b) What happens to the mass of the object if it is given a negative charge by induction? Explain.

REASONING AND SOLUTION A metallic rod is given a positive charge by the process of induction as illustrated in Figure 18.8.

a. The metallic object becomes positive because, during the induction process, electrons are forced from the object to the earth. The mass of the object will decrease by an amount equal to the mass of the electrons that left the metallic object.

b. The metallic object becomes negative because, during the induction process, electrons are pulled onto the object from the earth. The mass of the object will increase by an amount that is equal to the mass of the "excess" electrons that are pulled onto the object.

6. A proton and an electron are held in place on the x axis. The proton is at $x = -d$, while the electron is at $x = + d$. They are released simultaneously, and the only force that affects their motions is the electrostatic force of attraction that each applies to the other. Which particle reaches the origin first? Give your reasoning.

REASONING AND SOLUTION A proton and electron are held in place on the x axis. The proton is at $x = -d$, while the electron is at $x = +d$. They are released simultaneously, and the only force that affects their motions is the electrostatic force of attraction that each applies to the other. According to Newton's third law, the force F_{pe} exerted on the proton by the electron is equal in magnitude and opposite in direction to the force F_{ep} exerted on the electron by the proton. In other words, $F_{pe} = -F_{ep}$. According to Newton's second law, this equation can be written
$$m_p a_p = -m_e a_e$$
(1)

where m_p and m_e are the respective masses and a_p and a_e are the respective accelerations of the proton and the electron. Since the mass of the electron is considerably smaller than the mass of the proton, the acceleration of the electron at any instant must be considerably greater than the acceleration of the proton at that instant in order for Equation (1) to hold. Since the electron has a much greater acceleration than the proton, it will attain greater velocities than the proton and, therefore, reach the origin first.

11. Three point charges are fixed to the corners of a square, one to a corner, in such a way that the net electric field at the empty corner is zero. Do these charges all have (a) the same sign and (b) the same magnitude (but, possibly, different signs)? Justify your answers.

REASONING AND SOLUTION   Three point charges are fixed to the corners of a square (side length L and diagonal length d), one to a corner, in such a way that the net electric field at the empty corner is zero.
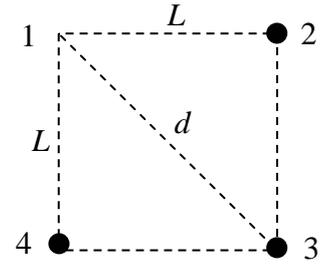
a. Each charge produces an electric field that is radially toward (positive charges) or radially away from (negative charges) the charge. The net electric field at corner 1 is the vector sum of the electric fields due to each of the charges at corners 2, 3 and 4. Since the electric field at corner 1 is zero, then the charges in corners 2 and 4 must have the same sign and have the opposite sign to the charge in corner 3. Suppose, for example, that the charge in corner 3 is positive. Then the electric field at corner 1 due to the charge in corner 3 must point away from corner 3 along the diagonal. In order for the vector sum of the fields due to all three charges to vanish at corner 1, the electric field due to the charge in corner 2 must cancel the *x* component of the electric field due to the charge in corner 3. Likewise, the electric field due to the charge in corner 4 must cancel the *y* component of the electric field due to the charge in corner 3. Thus, at corner 1, the electric field due to the charge at corner 2 must point to the right, while the electric field due to the charge in corner 4 must point toward the bottom of the page. The charges in corners 2 and 4, therefore, must be negative. Similar arguments apply if the charge in corner 3 is negative; we can then deduce that the charges in corners 2 and 4 must be positive. We conclude, therefore, that the three charges *cannot* all have the same sign.

b. Let all three charges have magnitude $|q|$. Then the magnitude of the electric field at corner 1 due to the charge in corner 3 is given by Equation 18.3:

$$E_{13} = \frac{k|q|}{d^2} = \frac{k|q|}{2L^2}$$

As discussed in part (a), this electric field vector points along the diagonal. Therefore, its *x* and *y* components have the following magnitudes

$$(E_{13})_x = \frac{k|q|}{2L^2} \cos 45° = \frac{k|q|}{2L^2} \left(\frac{\sqrt{2}}{2} \right) = \frac{k|q|}{(2\sqrt{2})L^2}$$

$$(E_{13})_y = \frac{k|q|}{2L^2} \sin 45° = \frac{k|q|}{2L^2} \left(\frac{\sqrt{2}}{2} \right) = \frac{k|q|}{(2\sqrt{2})L^2}$$

In order for the net electric field at corner 1 to be zero, the charges in corners 2 and 4 must have the opposite sign to the charge in corner 3 [as discussed in part (a)], and the electric field at corner 1 due to the charge in corner 2 must be equal to $-\left(E_{13} \right)_x$.

Similarly, the electric field at corner 1 due to the charge in corner 4 must be equal to $-(E_{13})_y$. If the charges in all three corners have the same magnitude, however, the

electric field at corner 1 due to the charge at corner 2 and the electric field at corner 1 due to the charge at corner 4 have magnitudes given by Equation 18.3 as

$$E_{12} = E_{14} = \frac{k|q|}{L^2}$$

Clearly, $E_{12} \neq \left(E_{13}\right)_x$ and $E_{14} \neq \left(E_{13}\right)_y$; therefore, all three charges *cannot* have the same magnitude.

CHAPTER 18 | *ELECTRIC FORCES AND ELECTRIC FIELDS*

Samples of solutions to Problems from chapter 18 Cutnell & Johnson 7E

5. ssm Consider three identical metal spheres, A, B, and C. Sphere A carries a charge of $+5q$. Sphere B carries a charge of $-q$. Sphere C carries no net charge. Spheres A and B are touched together and then separated. Sphere C is then touched to sphere A and separated from it. Last, sphere C is touched to sphere B and separated from it. (a) How much charge ends up on sphere C? What is the total charge on the three spheres (b) before they are allowed to touch each other and (c) after they have touched?

☐ **SSM** *REASONING* Identical conducting spheres equalize their charge upon touching. When spheres A and B touch, an amount of charge $+q$, flows from A and instantaneously neutralizes the $-q$ charge on B leaving B momentarily neutral. Then, the remaining amount of charge, equal to $+4q$, is equally split between A and B, leaving A and B each with equal amounts of charge $+2q$. Sphere C is initially neutral, so when A and C touch, the $+2q$ on A splits equally to give $+q$ on A and $+q$ on C. When B and C touch, the $+2q$ on B and the $+q$ on C combine to give a total charge of $+3q$, which is then equally divided between the spheres B and C; thus, B and C are each left with an amount of charge $+1.5q$.

SOLUTION Taking note of the initial values given in the problem statement, and summarizing the final results determined in the *Reasoning* above, we conclude the following:
  a.   Sphere C ends up with an amount of charge equal to $\boxed{+1.5q}$.

  b.   The charges on the three spheres before they were touched, are, according to the problem statement, $+5q$ on sphere A, $-q$ on sphere B, and zero charge on sphere C. Thus, the total charge on the spheres is $+5q - q + 0 = \boxed{+4q}$.

  c.   The charges on the spheres after they are touched are $+q$ on sphere A, $+1.5q$ on sphere B, and $+1.5q$ on sphere C. Thus, the total charge on the spheres is $+q + 1.5q + 1.5q = \boxed{+4q}$.

11. ssm Consult **Concept Simulation 18.1** at www.wiley.com/college/cutnell for insight into this problem. Three charges are fixed to an *x, y* coordinate system. A charge of $+18\,\mu C$ is on the *y* axis at *y* = + 3.0 m. A charge of $-12\,\mu C$ is at the origin. Last, a charge of $+45\,\mu C$ is on the *x* axis at *x* = + 3.0 m. Determine the magnitude and direction of the net electrostatic force on the charge at *x* = + 3.0 m. Specify the direction relative to the – *x* axis.

11. [SSM] *REASONING AND SOLUTION* The net electrostatic force on charge 3 at *x* = +3.0 m is the vector sum of the forces on charge 3 due to the other two charges, 1 and 2. According to Coulomb's law (Equation 18.1), the magnitude of the force on charge 3 due to charge 1 is

Figure 1

$$F_{13} = \frac{k\,|q_1||q_3|}{r_{13}^2}$$

where the distance between charges 1 and 3 is r_{13}.

According to the Pythagorean theorem, $r_{13}^2 = x^2 + y^2$. Therefore,

$$F_{13} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(18\times10^{-6}\ \text{C}\right)\left(45\times10^{-6}\ \text{C}\right)}{\left(3.0\ \text{m}\right)^2 + \left(3.0\ \text{m}\right)^2} = 0.405\ \text{N}$$

Charges 1 and 3 are equidistant from the origin, so that $\theta = 45°$ (see Figure 1). Since charges 1 and 3 are both positive, the force on charge 3 due to charge 1 is repulsive and along the line that connects them, as shown in Figure 2. The components of F_{13} are:

$$F_{13x} = F_{13}\cos 45° = 0.286\ \text{N} \qquad \text{and} \qquad F_{13y} = -F_{13}\sin 45° = -0.286\ \text{N}$$

The second force on charge 3 is the attractive force (opposite signs) due to its interaction with charge 2 located at the origin. The magnitude of the force on charge 3 due to charge 2 is, according to Coulomb's law ,

Figure 2

$$F_{23} = \frac{k\,|q_2||q_3|}{r_{23}^2} = \frac{k\,|q_2||q_3|}{x^2}$$

$$= \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(12\times10^{-6}\ \text{C}\right)\left(45\times10^{-6}\ \text{C}\right)}{\left(3.0\ \text{m}\right)^2}$$

$$= 0.539\ \text{N}$$

Since charges 2 and 3 have opposite signs, they attract each other, and charge 3 experiences a force to the left as shown in Figure 2. Taking up and to the right as the positive directions, we have
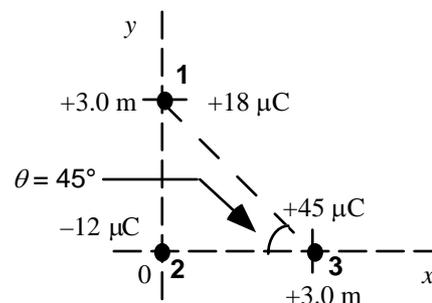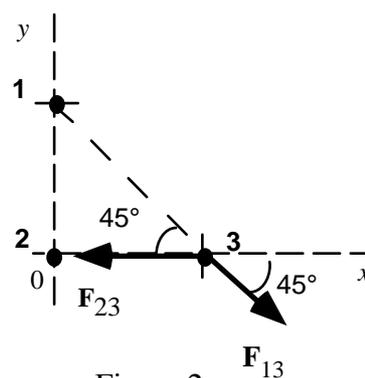
$$F_{3x} = F_{13x} + F_{23x} = +0.286 \text{ N} - 0.539 \text{ N} = -0.253 \text{ N}$$

$$F_{3y} = F_{13y} = -0.286 \text{ N}$$

Using the Pythagorean theorem, we find the magnitude of F_3 to be

$$F_3 = \sqrt{F_{3x}^2 + F_{3y}^2} = \sqrt{(-0.253 \text{ N})^2 + (-0.286 \text{ N})^2} = \boxed{0.38 \text{ N}}$$

The direction of F_3 relative to the $-x$ axis is specified by the angle ϕ, where

$$\phi = \tan^{-1}\left(\frac{0.286 \text{ N}}{0.253 \text{ N}}\right) = \boxed{49° \text{ below the } - x \text{ axis}}$$
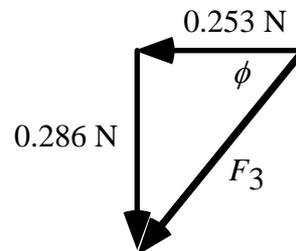
Figure 3

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39. ssm www A rectangle has a length of $2d$ and a height of d. Each of the following three charges is located at a corner of the rectangle: $+ q_1$ (upper left corner), $+ q_2$ (lower right corner), and $-q$ (lower left corner). The net electric field at the (empty) upper right corner is zero. Find the magnitudes of q_1 and q_2. Express your answers in terms of q.

39. $\boxed{\text{SSM}}$ $\boxed{\text{WWW}}$ *REASONING* The drawing shows the arrangement of the three charges. Let $\mathbf{E}_q$ represent the electric field at the empty corner due to the $-q$ charge. Furthermore, let $\mathbf{E}_1$ and $\mathbf{E}_2$ be the electric fields at the empty corner due to charges $+q_1$ and $+q_2$, respectively.
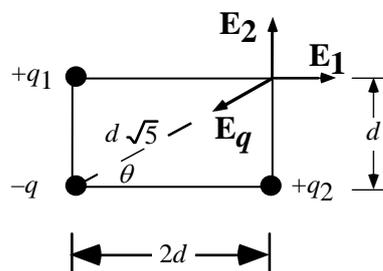
According to the Pythagorean theorem, the distance from the charge $-q$ to the empty corner along the diagonal is given by $\sqrt{(2d)^2 + d^2} = \sqrt{5d^2} = d\sqrt{5}$. The magnitude of each electric field is given by Equation 18.3, $E = k|q|/r^2$. Thus, the magnitudes of each of the electric fields at the empty corner are given as follows:

$$E_q = \frac{k|q|}{r^2} = \frac{k|q|}{\left(d\sqrt{5}\right)^2} = \frac{k|q|}{5d^2}$$

$$E_1 = \frac{k|q_1|}{(2d)^2} = \frac{k|q_1|}{4d^2} \quad \text{and} \quad E_2 = \frac{k|q_2|}{d^2}$$

The angle θ that the diagonal makes with the horizontal is $\theta = \tan^{-1}(d/2d) = 26.57°$. Since the net electric field E_{net} at the empty corner is zero, the horizontal component of the net field must be zero, and we have

$$E_1 - E_q \cos 26.57° = 0 \qquad \text{or} \qquad \frac{k|q_1|}{4d^2} - \frac{k|q|\cos 26.57°}{5d^2} = 0$$

Similarly, the vertical component of the net field must be zero, and we have

$$E_2 - E_q \sin 26.57° = 0 \qquad \text{or} \qquad \frac{k|q_2|}{d^2} - \frac{k|q|\sin 26.57°}{5d^2} = 0$$

These last two expressions can be solved for the charge magnitudes $|q_1|$ and $|q_2|$.

SOLUTION Solving the last two expressions for $|q_1|$ and $|q_2|$, we find that

$$|q_1| = \tfrac{4}{5}q \cos 26.57° = \boxed{0.716\ q}$$

$$|q_2| = \tfrac{1}{5}q \sin 26.57° = \boxed{0.0895\ q}$$

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73. **Concept Question** The drawing shows two situations in which charges are placed on the x and y axes. They are all located at the same distance from the origin O. Without doing any calculations, does the net electric field at the origin in part (a) have a magnitude that is greater than, less than, or equal to the magnitude of the field at the origin in part (b)? Justify your answer.

Problem The distance between each of the charges and the origin is 6.1 cm. For each of the situations shown in the drawing, determine the magnitude of the net electric field at the origin O. Check to see that your results are consistent with your answer to the Concept Question.
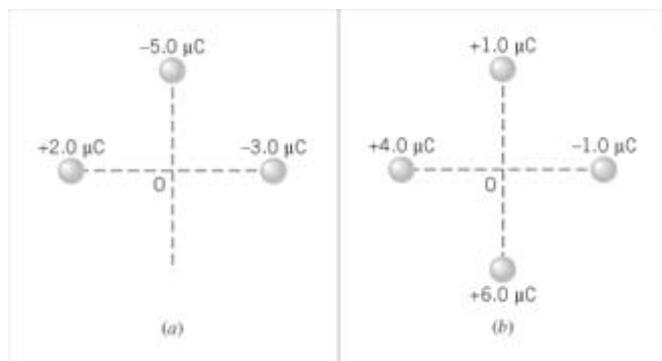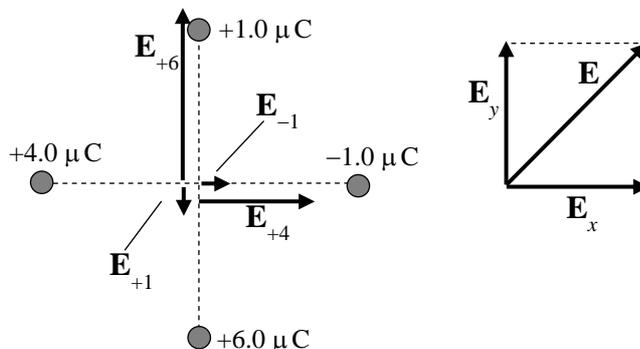

Part (a) of the drawing given in the text. The electric field produced by a charge points away from a positive charge and toward a negative charge. Therefore, the electric field $\mathbf{E}_{+2}$

produced by the +2.0 μC charge points away from it, and the electric fields $\mathbf{E}_{-3}$ and $\mathbf{E}_{-5}$ produced by the −3.0 μC and −5.0 μC charges point toward them (see the drawing that follows). The magnitude of the electric field produced by a point charge is given by Equation 18.3 as $E = k|q|/r^2$. Since the distance from each charge to the origin is the same, the magnitude of the electric field is proportional only to the magnitude $|q|$ of the charge. Thus, the x component $\mathbf{E}_x$ of the net electric field is proportional to 5.0 μC (2.0 μC + 3.0 μC). Since only one of the charges produces an electric field in the y direction, the y component $\mathbf{E}_y$ of the net electric field is proportional to the magnitude of this charge, or 5.0 μC. Thus, the x and y components are equal, as indicated in the right drawing, where the net electric field $\mathbf{E}$ is also shown.
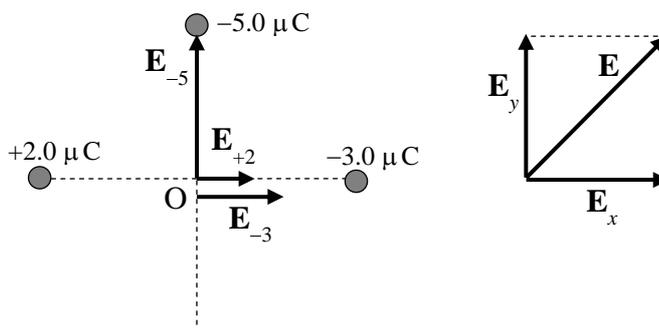
Part (*b*) of the drawing given in the text. Using the same arguments as earlier, we find that the electric fields produced by the four charges are shown in the left drawing. These fields also produce the same net electric field $\mathbf{E}$ as before, as indicated in the following drawing.

SOLUTION
Part (*a*) of the drawing given in the text. The net electric field in the x direction is

$$E_x = \frac{\left(8.99\times10^9 \ \mathrm{N\cdot m^2/C^2}\right)\left(2.0\times10^{-6}\ \mathrm{C}\right)}{\left(0.061\,\mathrm{m}\right)^2} + \frac{\left(8.99\times10^9 \ \mathrm{N\cdot m^2/C^2}\right)\left(3.0\times10^{-6}\ \mathrm{C}\right)}{\left(0.061\,\mathrm{m}\right)^2}$$

$$= 1.2\times10^7 \ \mathrm{N/C}$$

The net electric field in the *y* direction is

$$E_y = \frac{\left(8.99 \times 10^9 \ \text{N} \cdot \text{m}^2/\text{C}^2\right)\left(5.0 \times 10^{-6} \ \text{C}\right)}{\left(0.061 \ \text{m}\right)^2} = 1.2 \times 10^7 \ \text{N/C}$$

The magnitude of the net electric field is

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{\left(1.2 \times 10^7 \ \text{N/C}\right)^2 + \left(1.2 \times 10^7 \ \text{N/C}\right)^2} = \boxed{1.7 \times 10^7 \ \text{N/C}}$$

Part (*b*) of the drawing given in the text. The magnitude of the net electric field is the same as determined for part (a); $E = \boxed{1.7 \times 10^7 \ \text{N/C}}$.